

# Towards precise predictions for the Higgsstrahlung at a linear collider\*

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We report on progress in our work on obtaining high precision standard model predictions for the Higgsstrahlung reaction at a linear collider.

## 1. MOTIVATION

Main production mechanisms of the standard model (SM) Higgs boson in  $e^+e^-$  collisions at a linear collider [1] are the Higgsstrahlung reaction

$$e^+e^- \rightarrow ZH \quad (1)$$

and the  $WW$  fusion process

$$e^+e^- \rightarrow W^*W^* \rightarrow \nu_e\bar{\nu}_e H. \quad (2)$$

The cross section of (1) decreases as  $1/s$  while that of (2) grows as  $\ln(s/m_H^2)$ . Hence, while the Higgsstrahlung dominates the Higgs boson production at low energies, the  $WW$  fusion process overtakes it at higher energies. For small values of the Higgs boson mass,  $m_H < 140$  GeV, as is the case considered in this talk, the Higgs boson dominantly decays into a  $b\bar{b}$  quark pair. As the  $Z$  boson of reaction (1) decays into a fermion-antifermion pair too, one actually observes the Higgsstrahlung through reactions with four fermions in the final state.

In the following we will concentrate on one specific four fermion channel relevant for detection of (1)

$$e^+(p_1) + e^-(p_2) \rightarrow \mu^+(p_3) + \mu^-(p_4) + \bar{b}(p_5) + b(p_6), \quad (3)$$

where the particle four momenta have been indicated in parentheses, and the corresponding bremsstrahlung reaction

$$e^+e^- \rightarrow \mu^+\mu^-\bar{b}b\gamma. \quad (4)$$

Reactions (3) and (4) should leave a particularly clear signature in a detector. To the lowest order of the SM, in the unitary gauge and neglecting the Higgs boson coupling to the electron, reactions (3) and (4) receive contributions from 34 and 236 Feynman diagrams, respectively. Typical examples of the Feynman diagrams of reaction (3) are depicted in Fig. 1. The Higgsstrahlung ‘signal’ diagram is shown in Fig. 1a, while the diagrams in Figs. 1b and 1c represent typical ‘background’ diagrams. The Feynman diagrams of reaction (4) are obtained from those of reaction (3) by attaching an external photon line to each electrically charged particle line.

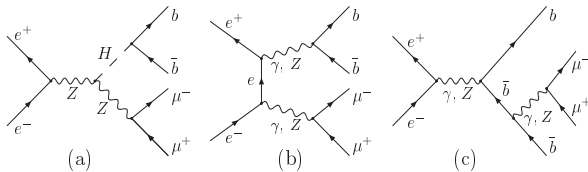


Figure 1. Examples of the Feynman diagrams of reaction (3): (a) the double resonance ‘signal’, (b) and (c) the ‘background’ diagrams.

In order to match the precision of data from a high luminosity linear collider, expected to be better than 1%, it is necessary to include radiative corrections in the SM predictions. At the

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moment, however, it is not feasible to include the complete  $\mathcal{O}(\alpha)$  corrections to a four-fermion reaction like (3) into a Monte Carlo (MC) generator. Therefore it seems to be natural to apply the double pole approximation (DPA), as it had been done before in literature in case of the  $W$ -pair production [2] at LEP2.

To the lowest order of the SM, the cross sections of reactions (3) and (4) can be computed with a program **ee4fg** [3]. On the basis of **ee4fg**, we have written a dedicated program **eezh4f** that eventually should include all the leading radiative corrections to the four-fermion reactions relevant for the Higgs boson production and decay through the Higgsstrahlung mechanism. However, up to now it includes the electroweak  $\mathcal{O}(\alpha)$  radiative corrections to the on-shell reaction (1) of [4] and the initial state radiation (ISR) QED corrections. No corrections to  $Z$  and Higgs decay widths, nor higher order ISR effects, nor any non-factorizable corrections are included in **eezh4f** yet.

## 2. OUTLINE OF THE CALCULATION

The amplitude of (1) is represented in the following way

$$M_{ZH}(p_1, \sigma_1, p_2, \sigma_2, k_Z, \lambda_Z, k_H) = \bar{v}(p_1, \sigma_1) \times F^\mu(p_1, p_2, k_Z, k_H) \varepsilon_\mu^*(k_Z, \lambda_Z) u(p_2, \sigma_2), \quad (5)$$

where  $p_1, p_2, k_Z, k_H$  and  $\sigma_1, \sigma_2, \lambda_Z$  are the on-shell four momenta and helicities of the external particles. Neglecting the electron mass, the amplitude  $F^\mu(p_1, p_2, k_Z, k_H)$  is decomposed in the following Lorentz covariant way

$$F^\mu(p_1, p_2, k_Z, k_H) = A_1 \gamma^\mu + A_2 \gamma^\mu \gamma_5 + (A_{31} p_1^\mu + A_{32} p_2^\mu) \not{k}_Z + (A_{41} p_1^\mu + A_{42} p_2^\mu) \not{k}_Z \gamma_5, \quad (6)$$

where we have changed a little the original notation with respect to that of Eq. (2.5) of [4]. The six invariant amplitudes that do not vanish in the limit  $m_e \rightarrow 0$  are now denoted by  $A_i$  and  $A_{ji}$ ,  $i = 1, 2, j = 3, 4$ . In order to fix normalization we give here the two amplitudes,  $A_i^{(0)}$ ,  $i = 1, 2$ , that are nonzero in the lowest order. They read

$$A_i^{(0)} = \frac{g_{HZZ}}{s - m_Z^2 + im_Z \Gamma_Z} a_i, \quad (7)$$

with the vector and axial-vector coupling of the  $Z$  to leptons

$$a_1 = \frac{4s_W^2 - 1}{4s_W c_W} e_W, \quad a_2 = \frac{e_W}{4s_W c_W} \quad (8)$$

and the  $Z$ -Higgs boson coupling  $g_{HZZ} = e_W m_Z / (s_W c_W)$ ; the electric charge  $e_W = (4\pi\alpha_W)^{1/2}$  and electroweak mixing angle  $\theta_W$  are fixed by

$$\alpha_W = \sqrt{2} G_\mu m_W^2 s_W^2 / \pi, \quad s_W^2 = 1 - m_W^2 / m_Z^2, \quad (9)$$

where  $m_W$  and  $m_Z$  are the physical masses of the  $W$  and  $Z$  boson. This choice, which is exactly equivalent to the  $G_\mu$ -scheme of [4], is in the program referred to as the ‘*fixed width scheme*’ (FWS). We have introduced a shorthand notation  $s_W = \sin \theta_W$  and  $c_W = \cos \theta_W$  in Eqs. (8–9).

Amplitudes  $A_i$ ,  $i = 1, 2$ , of Eq. (6) are split into the Born part  $A_i^{(0)}$  of Eq. (7) and the one loop corrections in the following way

$$A_i = A_i^{(0)} + \delta A_{i\text{QED}} + \delta A_i, \quad i = 1, 2, \quad (10)$$

where  $\delta A_{i\text{QED}}$  denotes the QED infra-red (IR) divergent and  $\delta A_i$  the IR finite part of the one-loop electroweak corrections. The IR divergent part  $\delta A_{i\text{QED}}$  is factored out and combined with the soft bremsstrahlung correction to (1) resulting in a QED initial state radiation correction factor  $C_{\text{QED}}^{\text{ISR}}$  [4] that can be split into a universal cut-off dependent and non-universal constant part

$$C_{\text{QED}}^{\text{ISR}}(s, E_{\text{cut}}) = C_{\text{QED}}^{\text{univ.}}(s, E_{\text{cut}}) + C_{\text{QED}}^{\text{non-univ.}}, \quad (11)$$

with

$$C_{\text{QED}}^{\text{univ.}}(s, E_{\text{cut}}) = \frac{e^2}{2\pi^2} \left[ \left( \ln \frac{s}{m_e^2} - 1 \right) \ln \frac{2E_{\text{cut}}}{\sqrt{s}} + \frac{3}{4} \ln \frac{s}{m_e^2} \right], \quad (12)$$

and

$$C_{\text{QED}}^{\text{non-univ.}} = \frac{e^2}{2\pi^2} \left( \frac{\pi^2}{6} - 1 \right). \quad (13)$$

The electric charge  $e = (4\pi\alpha_0)^{1/2}$  in Eqs. (12–13) is given in terms of  $\alpha_0$  in the Thomson limit.

The one-loop amplitudes

$$\delta A_i = \delta A_i(s, \cos \theta), \quad A_{ji} = A_{ji}(s, \cos \theta) \quad (14)$$

are complex functions of  $s = (p_1 + p_2)^2$  and  $\cos \theta$ ,  $\theta$  being the Higgs boson production angle with respect to the initial positron beam in the centre of mass system (CMS). They are computed numerically with **eezh4f** making use of **FF 2.0**, a package to evaluate one-loop integrals by G. J. van Oldenborgh [5].

As the computation of the one-loop electroweak amplitudes of Eq. (14) slows down the MC integration substantially, a simple interpolation routine has been written that samples the amplitudes at a few hundred values of  $\cos \theta$  and then the amplitudes for all intermediate values of  $\cos \theta$  are obtained by a linear interpolation. This gives a tremendous gain in the speed of computation, while there is practically no difference between the results obtained with the interpolation routine and without it.

The one-loop virtual and soft bremsstrahlung corrected matrix element squared of (3) in the double pole approximation (DPA) reads

$$|M_{\text{virt+soft}}^{\text{ISR}}|^2 = |M^{(0)}|^2 (1 + C_{\text{QED}}^{\text{univ.}}) \quad (15)$$

$$+ |M_{DPA}^{(0)}|^2 C_{\text{QED}}^{\text{non-univ.}} + 2\text{Re} \left( M_{DPA}^{(0)*} \delta M_{DPA} \right),$$

where  $M^{(0)}$  is the lowest order matrix element of (3) and the QED correction factors are given by Eqs. (12) and (13); the lowest order matrix element  $M_{DPA}^{(0)}$  and the one-loop correction  $\delta M_{DPA}$  in the double pole approximation for the  $Z$  and Higgs boson are given by

$$M_{DPA}^{(0)} = \bar{v}(p_1) \left( A_1^{(0)} \gamma_\mu + A_2^{(0)} \gamma_\mu \gamma_5 \right) u(p_2) \quad (16)$$

$$\times P_Z^\mu P_H$$

$$\delta M_{DPA} = \bar{v}(p_1) [\delta A_1 \gamma_\mu + \delta A_2 \gamma_\mu \gamma_5$$

$$+ (A_{31} p_{1\mu} + A_{32} p_{2\mu}) \not{k}_Z$$

$$+ (A_{41} p_{1\mu} + A_{42} p_{2\mu}) \not{k}_Z \gamma_5] u(p_2)$$

$$\times P_Z^\mu P_H. \quad (17)$$

The pole factors  $P_Z^\mu$  and  $P_H$  in Eqs. (16) and (17) are given by

$$P_Z^\mu = \frac{\bar{u}(k_4) (a_1 \gamma^\mu + a_2 \gamma^\mu \gamma_5) v(k_3)}{(p_3 + p_4)^2 - m_Z^2 + i m_Z \Gamma_Z},$$

$$P_H = \frac{g_{Hbb} \bar{u}(k_6) v(k_5)}{(p_5 + p_6)^2 - m_H^2 + i m_H \Gamma_H}, \quad (18)$$

with the  $Z$ - $\mu$  couplings  $a_1, a_2$  given by Eq. (8) and the Higgs-bottom-quark coupling  $g_{Hbb} = -e_W/(2s_W)(m_b/m_W)$ .

The projected momenta  $k_Z$  and  $k_i$ , of Eqs. (17) and (18) are obtained from the four momenta  $p_i$ ,  $i = 3, \dots, 6$ , of the final state fermions of reaction (3) with the following projection procedure.

First the on-shell momentum and energy of the Higgs and  $Z$  boson in the CMS are found

$$|\vec{k}_H| = \frac{\lambda^{1/2}(s, m_Z^2, m_H^2)}{2s^{1/2}}, \quad \vec{k}_H = |\vec{k}_H| \frac{\vec{p}_5 + \vec{p}_6}{|\vec{p}_5 + \vec{p}_6|},$$

$$E_H = \sqrt{|\vec{k}_H|^2 + m_H^2}, \quad \vec{k}_Z = -\vec{k}_H, \quad E_Z = \sqrt{s} - E_H.$$

Denote four momenta  $p_5$  and  $p_3$  of reaction (3) that are boosted to the rest frame of the Higgs and  $Z$ , respectively, by  $p'_5$  and  $p'_3$ . The projected four momenta  $k'_i$ ,  $i = 5, 6$ , in the rest frame of the Higgs are obtained with

$$|\vec{k}'_5| = \frac{\lambda^{1/2}(m_H^2, m_5^2, m_6^2)}{2m_H}, \quad \vec{k}'_5 = |\vec{k}'_5| \frac{\vec{p}'_5}{|\vec{p}'_5|},$$

$$\vec{k}'_6 = -\vec{k}'_5, \quad E'_i = \sqrt{|\vec{k}'_i|^2 + m_i^2}.$$

Similarly one obtains  $k'_3$  and  $k'_4$  in the rest frame of the  $Z$ .

Four momenta  $k'_i$ ,  $i = 3, \dots, 6$ , are then boosted back to the CMS giving the projected four momenta  $k_i$ ,  $i = 3, \dots, 6$  of  $\mu^+$ ,  $\mu^-$ ,  $\bar{b}$  and  $b$  that satisfy the necessary on-shell relations

$$k_3^2 = k_4^2 = m_\mu^2, \quad k_5^2 = k_6^2 = m_b^2,$$

$$(k_3 + k_4)^2 = m_Z^2, \quad (k_5 + k_6)^2 = m_H^2.$$

The actual value of  $\cos \theta$  in Eq. (14) is given by  $\cos \theta = k_H^3/|\vec{k}_H|$ . The described projection procedure is not unique. As the Higgs boson width is small, the ambiguity between different possible projections is mainly related to the off-shellness of the  $Z$  boson and is of the order of  $\alpha \Gamma_Z/(\pi m_Z)$ .

### 3. NUMERICAL RESULTS

In this section, we present some numerical results for (3) which will be one of the best detection

channels of the Higgsstrahlung reaction (1) at the linear collider.

The computation has been performed with a program `eezh4f` in the FWS with the following set of initial SM electroweak physical parameters [7]:

$$m_W = 80.423 \text{ GeV}, \quad \Gamma_W = 2.118 \text{ GeV}, \quad (19)$$

$$m_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.4952 \text{ GeV}, \quad (20)$$

$$G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad (21)$$

$$\alpha_0 = 1/137.03599976. \quad (22)$$

The charged lepton masses in MeV are

$$\begin{aligned} m_e &= 0.510998902, & m_\mu &= 105.658357, \\ m_\tau &= 1776.99 \end{aligned} \quad (23)$$

and the heavy quark masses in GeV are

$$m_c = 1.5, \quad m_t = 174.3, \quad m_b = 4.4. \quad (24)$$

The light quark masses in MeV are assumed at

$$m_u = 62, \quad m_d = 83, \quad m_s = 215, \quad (25)$$

which together with  $\alpha_s = 0.133$  reproduces the hadronic contribution to the running of the fine structure constant. The Higgs boson mass is assumed to be  $m_H = 115 \text{ GeV}$  and its width is calculated to the lowest order of the SM.

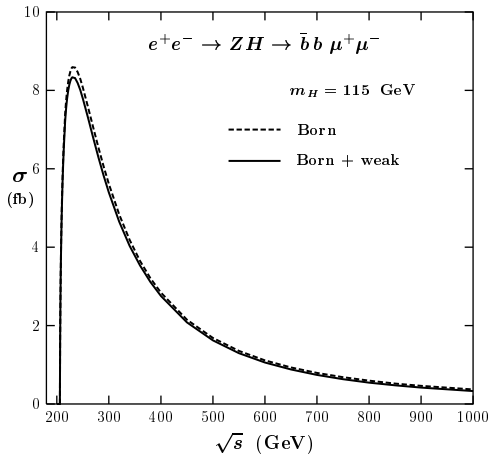


Figure 2. The ‘signal’ total cross section of reaction (3) in the narrow width approximation as a function of CMS energy.

In Fig. 2, we plot the total ‘signal’ cross section of (3) in the narrow width approximation, i.e. the cross section of (1) multiplied by the corresponding branching ratios, for the Higgs and Z boson, as a function of the CMS energy. The dashed curve shows the Born cross section while the solid curve shows the cross section including the one-loop electroweak corrections without the QED initial state radiation correction factor  $C_{\text{QED}}^{\text{ISR}}$  of Eq. (11). The corresponding relative correction is plotted in Fig. 3. The correction is of the order of a few per cent for  $\sqrt{s} < 500 \text{ GeV}$  and it grows almost linearly in the absolute value for higher energies, most probably due to large Sudakov logs that enter through the wave function normalization factors. Once the corrections to the Z and Higgs boson decay are included, these large corrections combined with the wave function renormalization of the Z and Higgs decay width should cancel against the renormalization constant of the Z and Higgs propagator.

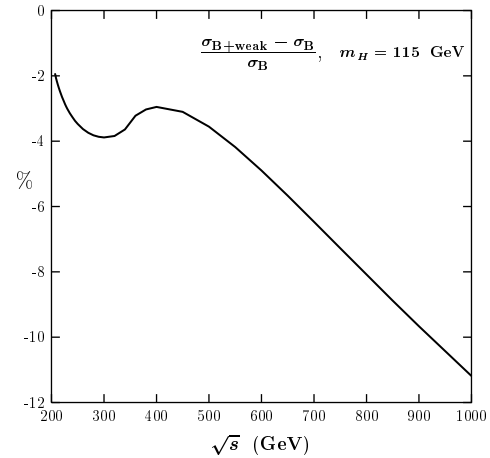


Figure 3. The relative corrections corresponding to that of Fig. 2

In Fig. 4, we compare results for the total cross sections of (3) as a function of the CMS energy, obtained with a complete set of the Feynman diagrams in the lowest order of the SM, including the Higgs boson (solid line) and without the Higgs boson exchange (dashed line). The Z–Higgs production signal is visible as the second bump of

the solid line, the first bump reflecting the double  $Z$  production resonance. The dotted line in Fig. 4 shows the  $\mathcal{O}(\alpha)$  corrected cross section including the QED ISR correction (12) applied to the complete set of Feynman diagrams of (3), the initial state hard photon radiation integrated over the full photon phase space and the electroweak radiative corrections to (1) in DPA. Fig. 5 focuses on the CMS energy region, where both resonances are present. We see that the photon radiation smears the resonances, as expected. The somewhat artificial large logarithmic correction of Fig. 3 has practically no visible effect in Figs. 4 and 5 as the  $\mathcal{O}(\alpha)$  is dominated by the QED ISR correction.

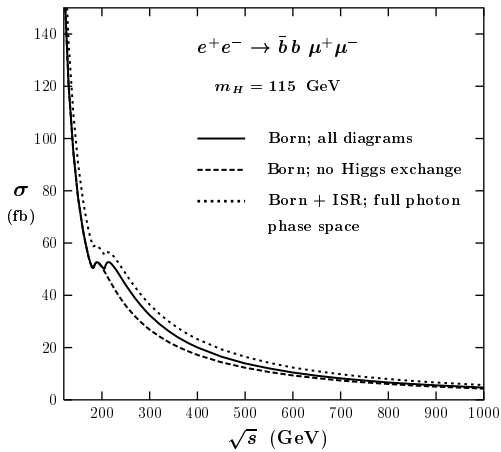


Figure 4. Total cross section of (3) as a function of CMS energy.

#### 4. SUMMARY AND OUTLOOK

The status of work on the high precision SM predictions for the Higgsstrahlung reaction at the linear collider has been reported. The QED ISR corrections to  $e^+e^- \rightarrow \mu^+\mu^-\bar{b}b$  have been included. The virtual one-loop weak corrections to (3) in the DPA have been included, too. Work on inclusion of the corrections to  $Z$  and Higgs decay widths and higher order corrections due to the QED ISR is in progress. The non-factorizable corrections should be included, too.

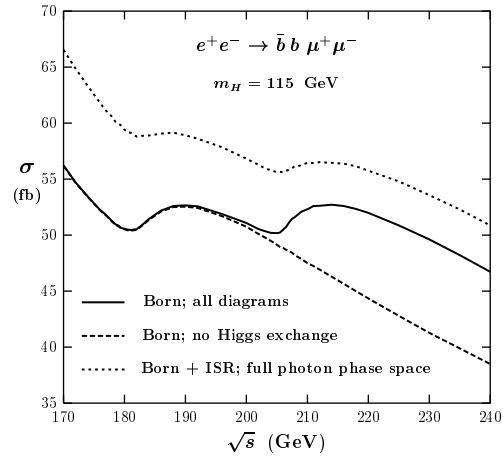


Figure 5. Total cross section of (3) in the vicinity of the  $ZZ$  and  $ZH$  resonances.

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